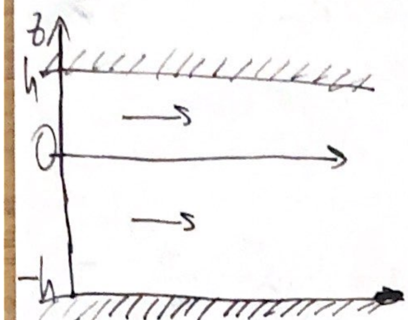
Плоское теч-е между парал-ми плоск-ми



1) Теч-е стац-ное $\frac{\partial}{\partial t}=0$

2) Теч-е парал-но оси Ox

3) Плоское теч-е $V_y=V_z=0$, $V_x=V(x,y,z)$

$$\rho\frac{dV}{dt}+\mathrm{grad}\,p=\mu\Delta V$$

Для несжимаемой жидкости:

$$\frac{\partial\rho}{\partial t}+\mathrm{div}(\rho V)=0\Rightarrow\mathrm{div}\,V=0$$

в плоском теч-и: $\Rightarrow\frac{\partial V}{\partial x}=0$

$$\frac{dV}{dt}=\frac{\partial V}{\partial t}+(V,\nabla)V;$$

$$\mathrm{grad}\,p=\mu\Delta V;\ \frac{\partial p}{\partial x}=\mu\left(\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\right)$$

$$\frac{\partial p}{\partial y}=\frac{\partial p}{\partial z}=0$$ (т.к. плоское теч-е и $V_y=V_z=0$) $\Rightarrow$

$\frac{\partial V}{\partial x}=0\Rightarrow V$ не зависит от $x\Rightarrow$

$\Rightarrow V(y,z)$

$\Rightarrow p$ не зависит от y и z, а зависит только от $x\Rightarrow\frac{\partial p}{\partial x}=\mathrm{const}$

$$\mu\left(\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)=\mathrm{const}$$

$$\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=\frac{1}{\mu}\cdot\frac{\partial p}{\partial x}$$

Будем искать частное реш-е V

$\frac{\partial V}{\partial x}=0\Rightarrow$ зависит только от $z\Rightarrow V(y,z)=V(z)$

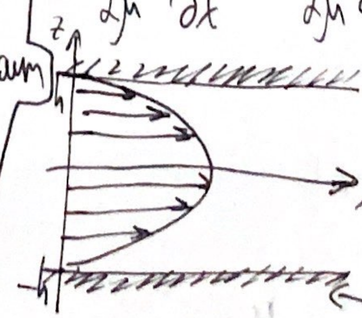
$$\frac{d^2V}{dz^2}=\frac{1}{\mu}\cdot\frac{\partial p}{\partial x};\ V=\frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot z^2+Az+B,$$

A, B - константы (из интегр.)

$V(h)=V(-h)=0$ - жидкость „прилипает" к пластинам.

$$\Rightarrow\begin{cases}0=\frac{1}{2\mu}\frac{\partial p}{\partial x}h^2+Ah+B\\ 0=\frac{1}{2\mu}\frac{\partial p}{\partial x}h^2-Ah+B\end{cases}\Rightarrow\begin{cases}B=-\frac{1}{2\mu}\frac{\partial p}{\partial x}h^2\\ A=0\end{cases}$$

$$V=\frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}z^2-\frac{1}{2\mu}\frac{\partial p}{\partial x}h^2=\frac{1}{2\mu}\frac{\partial p}{\partial x}(z^2-h^2)$$



$z=0$; $V=-\frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\cdot h^2$

$z=h$ $V=0$

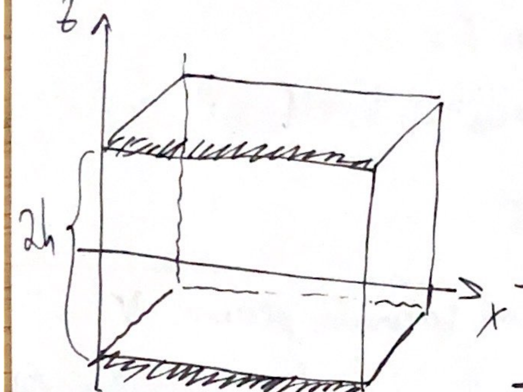
$z=-h$

← профиль ск-ти в зав-ти от z

⑤ для сохр-я напр-я движ-я жидкости (сонапр-на с Ox): $\frac{\partial p}{\partial x} < 0$ $(v > 0)$

$v > 0$

$\frac{\partial p}{\partial x} = \frac{p_1 - p_0}{l}$ μ_0 μ_1



$0 \le y \le b$, $-h \le z \le h$

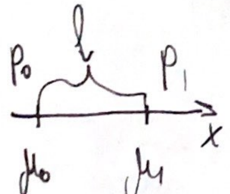
$Q = \int_0^b dy \int_{-h}^{h} v\,dz$

$\int_{-h}^{h} v\,dz = \int_{-h}^{h} \frac{1}{2\mu} \frac{\partial p}{\partial x} (z^2 - h^2)\,dz = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot \frac{z^3}{3}\Big|_{-h}^{h} -$

$- \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} h^2 z \Big|_{-h}^{h} = \frac{1}{2\mu} \frac{\partial p}{\partial x} \cdot \frac{h^3}{3} - \frac{1}{\mu} \cdot \frac{\partial p}{\partial x} \cdot h^3 =$

$= -\frac{2}{3\mu} \frac{\partial p}{\partial x} h^3$; $Q = -\frac{2}{3\mu} \cdot \frac{\partial p}{\partial x} h^3 b$

$\overline{v} = \frac{Q}{S} = \frac{-\frac{2}{3\mu} \frac{\partial p}{\partial x} h^3 l}{2hb} = -\frac{1}{3\mu} \frac{\partial p}{\partial x} \cdot h^2$



$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 v}{\partial z^2}$

$\frac{\partial p}{\partial x} = \frac{p_1 - p_0}{l}$

$\frac{\partial p}{\partial x} = \frac{-\frac{3\mu}{2} Q}{h^3 b} = -\frac{3}{2} \frac{\mu Q}{h^3 b} \Rightarrow \Delta p = p_1 - p_0 = -\frac{3}{2} \frac{\mu Q l}{2 h^3 b}$

$\frac{\partial p}{\partial x} = -\frac{3 \mu \overline{v}}{h^2} \Rightarrow \Delta p = p_1 - p_0 = \frac{3 \mu \overline{v} l}{h^2}$

$v = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} (z^2 - h^2) + \cancel{u}(y, z)$